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EXPONENTIAL BACKORDER COSTS AND CONTINUOUS LEAD TIMES FOR THE (M,T) INVENTORY MODEL SERIES 1

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Keywords: Exponential distribution, Continuous lead times, normal distribution, gamma distribution, stockout probability, backorder costs and inventory costs.

Abstract

The inventory model (M,T) with exponential backorder costs and continuous lead time is the model considered in this paper. At review time the stock is ordered to bring it to level M. The (M,T) model is derived from the inventory costs of inventory model (n,Q,R,T), when at review time a multiple of Q, n = 1,2---, is ordered, the lead time is assumed to follow a gamma distribution and demand during lead time is assumed to be a normal variate. The backorder cost is exponential, C_β (t)= b_1 exp (b_2 t).

Literature Review

Zipkin (2006), treats both fixed and random lead times and examiners both stationary and limiting distributions under different assumptions.

Bertsimas (1999) in his paper probabilistic service level guarantee in make – to – stock, considered both linear and quadratic inventory costs and backorder costs.

Pritibhushan Sinha (2008) in his paper a note on Bernornlli Demand inventory model presents a single – item, continuous monitoring inventory model with probabilistic demand for the item and probabilistic lead time of order replacement.

Hadley and Whitin (1922) extensively developed the inventory model (M,T) for constant lead times and linear backorder cost.

(n Q,R,T) Model, Exponential Backorder Costs.

(n Q,R,T) stands for the model in which at review time, the inventory position or the amount on hand plus on order at review time is less than or equal to R and the quantity ordered is a multiple of Q.

$C_\beta(t)$ is the expected backorder costs where t is the length of time of a backorder and is given by $C_\beta(t) = b_1 \exp(b_2 t)$ t > 0

Demand follows a normal distribution and $\sigma^2 t$ is the variance of demand over a period t. If the inventory position of the system immediately after review at time t, is R + Y, the expected backorder cost at time t, is

$$= \frac{1}{Q} \int_O^Q D \int_O^L D \int_O^t \frac{C_{\beta\,(t-Z)}}{\sqrt{\sigma^2 t}}\, g\left(\frac{R+Y-D_Z}{\sqrt{\sigma^2 l}}\right) \; dzdtdy$$

$Where\ \frac{1}{\sqrt{\sigma^2 l}}\, g\left(\frac{R+Y-D_Z}{\sqrt{\sigma^2 l}}\right)$ is the normal distribution (1)

Similarly the expected backorder costs at time t+L+T

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$$= \frac{1}{Q}\int_O^Q D \int_{O^O}^{L+T} D \int_O^t \frac{C_{\beta\,(t-Z)}}{\sqrt{\sigma^2 t}}\, g\left(\frac{R+Y-D_Z}{\sqrt{\sigma^2 t}}\right) \; dzdtdy \qquad (2)$$

$C_\beta(L-z) = b_1 \exp\big(b_2(L-z)\big)$

Let $G_1(Q,R,L) = \frac{1}{Q}\int_O^Q D \int_O^L D \int_O^t \frac{b_1 \exp b_2(L-Z)}{\sqrt{\sigma^2 t}}\, g\left(\frac{R+Y-D_Z}{\sqrt{\sigma^2 t}}\right) \; dzdtdy \qquad (3)$

Integrating $G_1(Q, R, L)$ with respect to Q and rearranging we have

$$G_2(R,L) = \int_O^L \frac{b_1}{b_2}\left(\exp - \left(\frac{b_2 R}{D} - b_2 t - \frac{\sigma^2 t\, b_2^2}{2D^2}\right)\left(F\left(\frac{R-\frac{\sigma_2^2 t - Dt}{D}}{\sqrt{\sigma^2 t}}\right) - F\left(\frac{R-Dt}{\sqrt{\sigma^2 t}}\right)\right)\right. \qquad dt$$

(4)

Then $G_1(R,L) = \frac{1}{Q}\left(G_2(R,L) - G_2(R+Q,L)\right) \qquad (5)$

Re-arranging the exponential terms of $G_2(R,L)$

$$G_2(R,L) = D\int_O^L \frac{b_1}{b_2}\exp\left(t\left(\frac{\sigma^2 b_2^2}{2D^2} + b_2\right) - \frac{b_2 R}{D}\right)\left(F\left(\frac{R-\frac{\sigma^2 b_2 t}{D} - Dt}{\sqrt{\sigma^2 t}}\right)\right.$$

Integrating by parts

$$\frac{1}{D}\, G_2\,(R,L) = \frac{b_1}{b_2}\left(\frac{2D^2}{\sigma^2 b_2^2 + 2D^2 b_2}\right)\left[exp\left(t\left(\frac{\sigma^2 b_2^2}{2D^2} + b_2\right) - \frac{b_2 R}{D}\right)\right.$$

$$\left.\left(F\left(\frac{R - \frac{\sigma^2 t\, b_2}{D} - Dt}{\sqrt{\sigma^2 t}}\right)\right)\right]_0^L \frac{b_1}{2b_2}\left(\frac{2D^2}{\sigma^2 b_2^2 + 2D^2 b_2}\right)$$

$$\int_O^L \exp\left(t\left(\frac{\sigma^2 b_2^2}{2D^2} + b_2\right) - \frac{b_2 R}{D}\right)\exp - \frac{1}{2}\left(\frac{R - \frac{\sigma^2 b_2 z}{D} - Dz}{\sqrt{\sigma^2 t}}\right)$$

$$\left(\frac{D^2 + \sigma^2 b_2}{D\sqrt{\sigma^2 t}} + \frac{R}{\sigma t^3/_2}\right) dz - \frac{b_1}{b_2}\int_O^L F\left(\frac{R-Dt}{\sqrt{\sigma^2 t}}\right) dt$$

Let

$$Z_n(x,T) = \int_O^T t^n \exp - \frac{1}{2}\left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right) dt$$

And the fact that

$$Z_{n+1}(x,T) = \frac{2\sigma^2 T^{n+1}}{D^2\sqrt{2\pi\sigma^2 T}}\exp - \frac{1}{2}\left(\frac{x-Dt}{\sqrt{\sigma^2 T}}\right)^2 - \frac{\sigma^2(2n+1)}{2D^2}\, Z_n(x,T) + \frac{x^2}{D^2}\, Z_{n-1}(x,T)$$

$$Z_{n+1}(x,T) = \frac{-2\sigma^2 T}{D^2\sqrt{2\pi\sigma^2 T}}\exp - \frac{1}{2}\left(\frac{x-DT}{\sqrt{\sigma^2 T}}\right)^2 + \frac{\sigma^2}{D^2} Z_O(x,T) + \frac{x^2}{D^2}\, Z_{n-1}(x,T)$$

We obtain

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$$G_2(R,L) = \frac{2D^3 b_1}{(\sigma^2 b_2^3 + 2D^2 b_2^2)}\, exp\left(L\left(\frac{\sigma^2 b_2^2 + 2D^2 b_2}{2D^2}\right) exp\left(\frac{-b_2}{D}\right)\right.$$
$$F\left(\frac{R - DL\left(1 + \frac{\sigma^2 b_2}{D^2}\right)}{\sqrt{\sigma^2 L}}\right) - \frac{b_1}{b_2}\, F\left(\frac{R-DL}{\sqrt{\sigma^2 L}}\right)\left[DL - R - \frac{\sigma^2}{2D} + \frac{D}{b_2}\right]$$
$$-\frac{\sigma^4 b_1 b_2^2}{2D(\sigma^2 b_2^2 + 2D^2 b_2) b_2}\, exp\left(\frac{2RD}{\sigma^2}\right) F\left(\frac{R+DL}{\sqrt{\sigma^2 L}}\right) - \frac{2\sqrt{\sigma^2 L}\, b_1}{b_2}\, g\left(\frac{R-DL}{\sqrt{\sigma^2 L}}\right)$$

Hence

$$G_1(Q,R,L) = \big((G_2(R,L) - G_2(R+Q,L)\big)/Q$$

Expected backorder cost at t + L, averaging over the states of Y

$$= \frac{1}{Q}\left(G_2(R,L) - G_2(R+Q,L)\right)$$

And the expected backorder cost at t + L + T averaging over the states of y

$$= \frac{1}{Q}\left(G_2(R,L+T) - G_2(R+Q,L+T)\right)$$

Hence the expected backorder cost per year G_3 (Q,R,T)

$$G_3(Q,R,T) = \frac{1}{Q}\left(G_2(R,L+T) - G_2(R,L) - G_2(R+Q,L+T) + G_2(R+Q,L)\right) \quad (7)$$

Let B(Q,R,T)be the expected number of backorders, at any point in time.

At anytime t + L + ∈ between t + L and t + L + T, the expected number of backorders on the book when the inventory position was R + y immediately after review at time t

w = R + y

$$B(Q,R,T) = \frac{1}{QT}\int_Q^{R+Q}\int_L^{L+T}\int_\omega^\infty (x-w)\, g(x,Dt)\; dx dt dw$$

Nothing that

$$\int_w^\infty (x-w)\, g(x,Dt)\, dx = \sigma^2 t\, g(w,Dt) - (w-Dt)\, F\left(\frac{w-Dt}{\sqrt{\sigma^2 t}}\right)$$

Substituting in B$(Q,R,T)$and simplifying

We have

$$B(Q,R,T) = \frac{1}{QT}\int_Q^{R+Q}\int_L^{L+T}\left(\sigma^2 t\, g(w-Dt) - (w-Dt)\, F\left(\frac{w-Dt}{\sqrt{\sigma^2 t}}\right)\right) dt\, dw$$
$$-\frac{1}{QT}\int_Q^{R+Q}\int_O^{L}\left(\sigma^2 t\, g(w,Dt) - (w-Dt)\, F\left(w,Dt\right)\right) dt\, dw$$

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Noting that

$$\int_o^T g\,(x, Dt)dt = \frac{1}{D}\left(F\left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right) - exp\left(\frac{2Dx}{\sigma^2}\right)F\left(\frac{x+Dt}{\sqrt{\sigma^2 t}}\right)\right)$$

$$\int_o^T t\,F\left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right)dt = \frac{1}{2}\left(T^2 - \frac{x^2}{D^2} - \frac{2\sigma^2 x}{D^3} - \frac{3\sigma^4}{2\sigma^4}\right)F\left(\frac{x-DT}{\sqrt{\sigma^2 T}}\right)$$

$$+\frac{\sqrt{\sigma^2 T}}{2D^2}\left(DT + \frac{3\sigma^2}{D} - x\right)\frac{1}{\sqrt{2\pi\sigma^2 T}}\exp -\frac{1}{2}\left(\frac{x-DT}{\sqrt{\sigma^2 T}}\right)^2 - \frac{\sigma^2}{2D^3}$$

$$\left(x - \frac{3\sigma^2}{2D}\right)exp\left(\frac{2Dx}{\sigma^2}\right)F\left(\frac{x+DT}{\sqrt{\sigma^2 T}}\right)$$

$$\int_o^T F\left(\frac{x-DT}{\sqrt{\sigma^2 T}}\right)dt = \left(T - \frac{x}{D} - \frac{\sigma^2}{2D^2}\right)F\left(\frac{x-DT}{\sqrt{\sigma^2 T}}\right) + \frac{\sqrt{\sigma^2 T}}{D\sqrt{2\pi}}\exp -\frac{1}{2}\left(\frac{x-DT}{\sqrt{\sigma^2 T}}\right)^2 + \frac{\sigma^2}{2D^2}$$

$$exp\left(\frac{2Dx}{\sigma^2}\right)F\left(\frac{x+DT}{\sqrt{\sigma^2 T}}\right)$$

We obtain

$$B(Q,R,T) = \frac{1}{QT}\,(G_4(R+L,T) - G_4(R,L) - G_4(R+Q,L+T) + G_4(R+Q,L)) \qquad (8)$$

Let POR be the probability of a stock out, at anytime between t + L and t + L + T, that is the probability that demand exceeds R + y at t + L + ε.

$$= \int_{R+y}^{\infty} g\,(x, D\,(L+\varepsilon)\,dx.$$

Probability of a stock out

$$\text{POR} \;= \frac{1}{Q}\int_L^{L+T}\int_{R+y}^{\infty} g\,(x, D\varepsilon)dx\,d\varepsilon$$

Averaging the states of y and integrating with respect to x

$$\text{POR} \;= \frac{1}{Q}\int_o^Q\int_L^{L+T} F\,(R+y, D\varepsilon)d\epsilon\,dy$$

$$Let\ G_5(R,T) = \int_R^{\infty}\int_o^T F\left(\frac{x-Dt}{\sqrt{\sigma^2 t}}\right)dt$$

$$= \left(\frac{(R-DT)^2}{2D} + \frac{\sigma^2 R}{2D^2} + \frac{\sigma^4}{4D^3}\right)F\left(\frac{R-DT}{\sqrt{\sigma^2 T}}\right) + \frac{\sqrt{\sigma^2 T}}{2}\left(T - \frac{\sigma^2}{D^2} - \frac{R}{D}\right)$$

$$g\left(\frac{R-DT}{\sqrt{\sigma^2 T}}\right) - \frac{\sigma^4}{4D^3}\,F\left(\frac{R+DT}{\sqrt{\sigma^2 T}}\right)exp\left(\frac{2DR}{\sigma^2}\right)$$

Hence

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$$\text{POR} = \frac{1}{Q}\, G_5(R, L+T) - G_5(R,L) - G_5(R+Q, L+T) + G_5(R+Q, L))$$

Where $G_4(R,T)$

$$G_4(R,T) = \left(\frac{D^2T^3}{6} - \frac{\sigma^4 R}{4D^3} - \frac{DT^2R}{2} - \frac{\sigma^2R^2}{4D^2} + \frac{\sigma^2T^2}{4} + \frac{TR^2}{2} - \frac{R^3}{6D} - \frac{\sigma^6}{8D^4}\right)$$

$$F\left(\frac{R-DT}{\sqrt{\sigma^2 T}}\right) + \left(\frac{DT^{5/2}\sigma}{6} - \frac{\sigma T^{3/2}R}{3} + \frac{\sigma T^{1/2}R^2}{6D} + \frac{\sigma^3T^{3/2}}{12D} + \frac{\sigma^3T^{1/2}R}{4D^2}\right.$$

$$\left. + \frac{\sigma^5T^{1/2}}{4D^3}\right) g\left(\frac{R+DT}{\sqrt{\sigma^2T}}\right) + \frac{\sigma^6}{8D^4}\exp\left(\frac{2DR}{\sigma^2}\right) F\left(\frac{R+DT}{\sqrt{\sigma^2T}}\right)$$

Putting the various costs together we have the inventory costs excluding the cost dependent on number of stock outs to be for (Q, M, T)

$$R = \frac{Rc}{T} \quad \frac{S.POR}{1} + \frac{hc}{2}\left(\frac{Q}{2} + M - DL - \frac{DT}{2}\right) + \frac{hc}{QT}(G_4(M, T+L)$$

$$-G_4(M,L) - G_4(M+Q, T+L) + G_4\big(M+Q, L)\big) + \frac{1}{QT}(G_2(M, T+L) - G_2(M,L)$$

$$-G_2(M+Q, T+L) + G_2(M+Q, L)) \tag{9}$$

Lim POR = 1

Q ⟶ 0

$$Lim\ \frac{1}{Q} G_4(M,Q,T) = -\left(\frac{\sigma^4 - 2D^4T^2}{4D^3} + \frac{\sigma^2 - 2D^2T}{2D^2} + \frac{R^2}{2D}\right) F\left(\frac{R-DT}{\sqrt{\sigma^2T}}\right)$$

Q ⟶ 0

$$+\frac{1}{2}\left(\sigma T^{3/2} - \frac{\sigma^3 T^{\frac{1}{2}}}{D^2} - \frac{\sigma T^{\frac{1}{2}}R}{D}\right) g\left(\frac{R-DT}{\sqrt{\sigma^2T}}\right) - \frac{\sigma^4}{D^3} exp\ \left(\frac{2DR}{\sigma^2}\right)\ F\left(\frac{R-DT}{\sqrt{\sigma^2T}}\right) \tag{10}$$

Denoted – $G_5\ (M,T)$

$$Lim\ \ \frac{1}{Q}\ G_2\left(\frac{M+Q,L}{u}\right) = \frac{-2D^2b_1}{b_2(\sigma^2b_2^2+2D^2b_2)}\ exp\ \left[T\left(\frac{(\sigma^2b_2^2+2D^2b_2)}{2D^2}\right) - \frac{b_2}{D}\ M\right]$$

$$F\left(\frac{\left(M - T\left(D + \frac{\sigma^2b_2}{D}\right)\right)}{\sqrt{\sigma^2T}}\right) + \frac{b_1}{b_2}(M-DT)\ exp - \frac{1}{2}\left(\frac{M-DT}{\sqrt{\sigma^2T}}\right) + \frac{b_1}{b_2}$$

$$F\left(\frac{M-DT}{\sqrt{\sigma^2T}}\right) - \frac{\sigma^2b_2^2b_1}{b_2(\sigma^2b_2^2 + 2D^2b_2)} exp\ \left(\frac{2DM}{\sigma^2}\right)\ F\left(\frac{M+DT}{\sqrt{\sigma^2T}}\right)$$

Denoted - $G_6\ (M,T)$ (11)

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The inventory cost for model (M,T) when the cost is an exponential function of the length of a backorder for fixed lead times is T,

$$C = (Rc+s) + hc\left(M - DL - \frac{DT}{2}\right) + \frac{hc}{T}(G_5(M,T+L - G_5(M,L)$$

$$-\frac{1}{T}(G_6(M,L+T) - G_6(M,L_1)\Big) \quad (12)$$

The inventory costs when the lead-times are continuous random variables are obtained by averaging the cost for the fixed lead times over the states of the lead times.

$$Let G_7(R) = \int_o^a H(L)\,G_5(R,L)dL \quad (13)$$

$$G_8(R,T) = \int_o^a H(L)\,G_5(R,L+T)dL \quad (14)$$

$$Let G_9(R) = \int_o^a G_6(M,L)H(L)dL \quad (15)$$

$$G_{10}(R) = \int_o^a H(L)\,G_6(M,L+T)dL \quad (16)$$

Hence

$$G_6(M,L) = \frac{2Db_1}{b_2(\sigma^2 b_1^2 + 2D^2 b_2)}\left[L\left(\frac{\sigma^2 b_1^2 + 2D^2 b_2}{2D_2}\right) - \frac{b_2 M}{D}\right]F$$

$$\left[M - L\left(\frac{D+\sigma^2 b_2}{D}\right)\right] + \frac{b_1}{b_2}\frac{(M-DL)}{D\sqrt{\sigma^2 L}}\,g\left(\frac{(M-DL)}{D\sqrt{\sigma^2 L}}\right) - \frac{b_1}{b_2 D}F\frac{(M-DL)}{D\sqrt{\sigma^2 L}}$$

$$-\frac{\sigma^2 b^2{}_2 b_1 esp\left(\frac{2DM}{\sigma^2}\right)}{Db_4(\sigma^2 b_2{}^2 + 2D^2 b_2)}F\left(\frac{M-DL}{\sqrt{\sigma^2 L}}\right)$$

Multiplying H (L) where H(L) $=\frac{\propto^k L^{k-1} esp(-\propto L)}{\Gamma(k)}$ $\quad \propto, k, L > 0$

$$H(L)G_6(M,L) = \frac{2Db_1 \propto^k}{b_2(\sigma^3\, b_2^2 + 2D^2 b_2)}esp\left[L\left(\frac{\sigma^2 b^2{}_2 + 2D^2 b_2}{2D_2}\right) - \propto) - \frac{b_2 M}{D}\right]L^{k-1}$$

$$F\left(\frac{M - L\left(D + \frac{\sigma^2 b_2}{D}\right)}{\sqrt{\sigma^2 L}}\right) + \frac{b_1}{b_2}\frac{esp\,(-\propto L)}{D\sqrt{\sigma^2 L}}(ML^{k-1} - DL^{k-1})g\left(\frac{(M-DL)}{\sqrt{\sigma^2 L}}\right)$$

$$-\frac{b_1}{b_2 D}\,exp\,(-\propto L)L^{k-1}F\frac{(M-DL)}{\sqrt{\sigma^2 L}} - \frac{\sigma^2 b^2{}_2 b_1}{Db_2(\sigma^2 b^2{}_2 + 2D^2 b_2)\,\Gamma(K)}esp(-\propto L)L^{k-1}$$

$$exp\left(\frac{2DM}{\sigma^2}\right)F\left(\frac{M+DL}{\sqrt{\sigma^2 L}}\right)$$

Nothing that

$$\int_o^\infty H(L)\,g\left(\frac{n+DL}{\sqrt{\sigma^2 L}}\right)\frac{1}{\sqrt{\sigma^2 L}}$$

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$$= \int_o^\infty exp - (\propto l)\ \frac{L^{k-1}\propto^k}{\Gamma(k)}\, g\left(\frac{n-DL}{\sqrt{\sigma^2 L}}\right) dl$$

$$= \frac{\propto^k}{\sigma\sqrt{2\pi}\,\Gamma(k)} \int_0^\infty L^{k-3/2}\, exp\left(\frac{Dx}{\sigma^2}\right) exp\left(\frac{-x^2}{2\sigma^2 L} - \frac{L(2\propto\sigma^2 + D^2)}{2\sigma^2}\right) dL$$

$$= \frac{\propto^k}{\sigma\sqrt{2\pi}}\ \frac{exp\left(\frac{Dx}{\sigma^2}\right)}{\Gamma(k)} \left[2\left(\frac{x^2}{2\propto\sigma^2 + D^2}\right)^{\frac{1}{2}\left(k-\frac{1}{2}\right)} K_{k-\frac{1}{2}}\left(\frac{x}{\sigma^2}\left(2\propto^2 + D^2\right)^{\frac{1}{2}}\right)\right] \qquad (18)$$

If K is an in interger then

$$K_{k-\frac{1}{2}}(z) = K_{1\frac{1}{2}}(z) \sum_{j=0}^{k-1} \frac{(k+i-1)}{j!\,(k-i-1)} (2z)^{-j}$$

$$Where\ k_{\frac{1}{2}}(z) = \frac{\sqrt{\pi}}{\sqrt{2}} (z)^{\frac{-1}{2}} exp(-z)$$

Hence $k_{k-\frac{1}{2}}(z) = \sqrt{\pi} \sum_{j=0}^{k-1} \frac{(k+j-1)!}{j!\,(k-j-1)!} (2z)^{-j-\frac{1}{2}} exp\,(-z)$

$$\int_0^\infty H(L)\, G_6(M,L) dL\ \ applying\ equation\ 17{,}18$$

$$and\ putting\ \varepsilon^2 = 2\left(\frac{\sigma^2 {b_2}^2 + 2D^2 b_2}{2D^2} - 2D\propto\right)$$

$$G_9(M) = \frac{2Db_1\propto^k esp}{2\sigma(\sigma^2 {b_2}^{\ 2} + 2D^2 b_2)} \frac{\left(M\left(D + \frac{\sigma^2 b_2}{D}\right) - \frac{b_2 M}{D}\right)}{\Gamma(k)\sqrt{2\pi}}$$

$$\sum_{z=1}^{k} \frac{(k-1)!\,(k-z)!}{(\sigma^2 {b_2}^{\ 2} + 2D^2 b_2 - 2D^2\propto)!} \left(2\left(D + \frac{\sigma^2 b_2}{D}\right)\left(\frac{R}{\varepsilon}\right)^{k-z+1/2} K_{k-z+1/2}^{\left(\frac{R\varepsilon}{\sigma^2}\right)}\right.$$

$$\left. + 2R\left(\frac{R}{\varepsilon}\right)^{k-z-1/2} K_{k-2-1/2}^{\left(\frac{RL}{\sigma^2}\right)}\right] \frac{esp\left(\frac{DR}{\sigma^2}\right)}{2\pi.2\sigma b_2 D\ \ \Gamma(k)}$$

$$\sum_{z=1}^{k} \frac{(k-1)!}{\propto^z\,(k-z)!} \left[2D\left(\frac{R}{\theta}\right)^{k-z+1/2} K_{k-z+1/2}^{\left(\frac{R}{\sigma^2}\right)} - 2R\left(\frac{R}{\theta}\right)^{k-z-1/2} K_{k-z-1/2}^{\left(\frac{R}{\sigma^2}\right)}\right]$$

$$+ \frac{\sigma^2 {b_2}^{\ 2} b_1 esp\left(\frac{DR}{\sigma^2}\right)\propto^k}{Db_2(\sigma^2 b_2 + 2D^2 b_2)\ \ \Gamma(k)}$$

$$\sum_{z=1}^{k} \frac{(k-1)!\,(k-z)!}{\propto^z\,(\sigma^2 b_2 + 2D^2 b_2 - 2D^2\propto)!} \left[2D\left(\frac{R}{\theta}\right)^{k-z+1/2} K_{k-z+1/2}^{\left(\frac{R}{\sigma^2}\right)} - 2R\right.$$

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$$\left(\frac{R}{\theta}\right)^{k-z-1/2} K_{k-z-1/2}^{\left(\frac{R}{\sigma^2}\right)}\Bigg]$$

From 7 substituting L + T for L we have

$$G_6(M.L+T) = \frac{2Db_1}{b_2(\sigma^2 b_2 + 2D^2 b_2)} esp\left[L\left(\frac{\sigma^2 b_2 + 2D^2 b_2}{2D^2}\right) - \frac{b_2 M}{D}\right]$$

$$F\left(\frac{M-(L+T)\left(D+\frac{\sigma^2 b_2}{\sqrt{\sigma^2(L+T)}}\right)}{\sqrt{\sigma^2(L+T)}}\right) + \frac{b_1}{b_2}\left(\frac{(M-DT)-DL}{D\sqrt{\sigma^2(L+T)}}\right)$$

$$g\left(\frac{M-D(L+T)}{\sqrt{\sigma^2(L+T)}}\right) - \frac{b_1}{Db_2} F\left(\frac{M-D(L+T)}{\sqrt{\sigma^2(L+T)}}\right) - \frac{\sigma^2 b_2{}^2 b_1 esp\left(\frac{2DM}{\sigma^2}\right)}{Db_2(\sigma^2 b_2{}^2 + 2D^2 b_2)}$$

$$F\left(\frac{M-D(L+T)}{\sqrt{\sigma^2(L+T)}}\right)$$

Multiplying by H(L)

$$H(L)G_{19}(M,L+T) = \frac{2Db_1 \propto^k L^{k-1}}{b_4(\sigma^2 b_4 + 2D^2 b_4)\ \ (k)} \text{esp}\left[T\left(\frac{\sigma^2 b_4 + 2D^2 b_4}{2D^2}\right) - \frac{b_4 M}{D}\right]$$

$$esp\left(L\ \frac{(\sigma^2 b_4 + 2D^2 b_4) - a}{2D^2}\right) F\left(\frac{M-(L+T)\left(D+\frac{\sigma^2 b_4}{D}\right)}{\sqrt{\sigma^2 (L+T)^{1/2}}}\right) + \frac{b_1}{b_4}$$

$$esp(-aL) \propto^k \frac{(M-DT)L^{K-1} - DL^K}{D\sqrt{\sigma^2(L+T)}} g\left(\frac{M-D(L+T)}{\sqrt{\sigma^2(L+T)}}\right) - \frac{b_1}{Db_4}$$

$$\frac{esp\ (-\propto L)\ L^{K-1}\propto^K}{\Gamma(K)} F\left(\frac{M-D(L+T)}{\sqrt{\sigma^2(L+T)}}\right) - \frac{\sigma^2 b_2{}^2 b_1 esp(-\propto L)L^{K-1}\propto^K}{Db_4(\sigma^2 b_4{}^2 + 2D^2 b_4)\ (k)} g\left(\frac{M-D(L+T)}{\sqrt{\sigma^2(L+T)}}\right)$$

$$\int_0^\propto H(L)\ G_6(M,L+T)dl\ \ applying\ equations\ 18,19$$

$$G_{10}(M,T) = \frac{2Db_1}{2\sigma\ \ \Gamma(k)} \propto^k esp\ b_4\ \ \frac{\left[T\left(\frac{\sigma^2 b_4{}^2 + 2D^2 b_4 - b_4 M + \propto T + \frac{DM}{\sigma^2}}{2D^2}\right)\right]}{(\sigma^2 b_4{}^2 + 2D^2 b_4)^2}$$

$$\sum_{j=0}^{k-1}(-T)\binom{k-1}{j}\sum_{z=1}^{k-j}\frac{(k-1-j)!}{\left(\frac{\sigma^2 b_2{}^2 + 2D^2(b_2 - \propto)}{2D^2}\right)}\ \frac{1}{(k-z)!}\left(2\left(D+\frac{\sigma^2 b_2}{D}\right)\right.$$

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$$\left(\frac{M}{\varepsilon}\right)^{k-j-z+1/2}\Bigg) K_{k-j-z+1/2}^{\left(\frac{M\varepsilon}{\sigma^2}\right)} + 2M\left(\frac{M}{\varepsilon}\right)^{k-j-z-1/2} K_{k-j-z-1/2}^{\left(\frac{M\varepsilon}{\sigma^2}\right)}$$

$$+\frac{2b_1 \propto^k esp\left(\frac{DM}{\sigma^2}\right) esp(\propto T)}{Db_2\ \Gamma(k)\sqrt{2\pi\sigma^2}}$$

$$\sum_{j=0}^{k-1}(-T)^i\binom{k-1}{j}\left(\frac{M}{\theta}\right)^{k-j+1/2} K_{k-j+1/2}^{\left(\frac{M\varepsilon}{\sigma^2}\right)} - \frac{b_1 \propto^k esp\left(\frac{DM}{\sigma^2}\right) + \propto T}{\sqrt{2\pi}\, Db_2\, \Gamma(k)}$$

$$\sum_{j=0}^{k-1}(-T)^i\binom{k-1}{j}\sum_{z=1}^{k-j}\frac{(k-1-j)!}{\propto^z (k-j-z)!}\left(2D\left(\frac{M}{\theta}\right)^{k-j-z+1/2} K_{k-j-z+1/2}^{\left(\frac{M\theta}{\sigma^2}\right)} +\right.$$

$$\left. 2M\left(\frac{M}{\theta}\right)^{k-j-z-1/2} K_{k-j-z-1/2}^{\left(\frac{M9}{\sigma^2}\right)}\right) - \frac{\sigma^2 b_2{}^2 b_1 \propto^k esp\left(\frac{DM}{\sigma^2} + \propto T\right)}{2\sigma D b_2\left(\sigma^2 b_2{}^2 + 2D^2 b_2\right)\ \Gamma(k)\ \sqrt{(2\pi)}}$$

$$\sum_{j=0}^{k-1}(-T)^i\binom{k-1}{j}\sum_{z=1}^{k-j}\frac{(k-1-j)!}{\propto^z (k-j-z)!}\left(-2D\left(\frac{M}{\theta}\right)^{k-j-z+1/2} K_{k-j-z+1/2}^{\left(\frac{M9}{\sigma^2}\right)}\right.$$

$$\left. +2M\left(\frac{M}{\theta}\right)^{k-j-z-1/2} K_{k-j-z-1/2}^{\left(\frac{M9}{\sigma^2}\right)}\right) \qquad (21)$$

Hence integrating over the states of L the inventory cost for model (M,T) when the lead times is continuous random variable is

$$C = \frac{(R_1 + S)}{T} + hc\left(M - \frac{DK}{\propto} - \frac{DT}{2}\right) + \frac{hc}{T}\left(G_8(R,T) - G_7(R) + \frac{1}{T}\left(G_{10}(R,T) - G_9(R)\right)\right)$$

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